PROPAGATION OF CRACKS AND TWINS ALONG ANISOTROPIC PLATES

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Let us consider a semi-infinite crack in an anisotropic plate whose thickness is 2h. It is assumed that the plane of the crack is parallel to the plate boundaries and that it is kept open by certain external stresses. The positioning of the crack relative to the plate boundaries is shown in Fig. 1; a plane problem is being considered. Let us study the propagation of the crack as a result of an increase in the external load.

A narrow equilibrium crack may be formally described [1] as an aggregate of equilibrium dislocations at the crack edges. The Burgers vectors of these dislocations are normal to the crack plane. This approach is identical with the method developed by Barenblatt [2].

If the dislocation density is denoted by $\rho(x)$, the integral equation of the crack equilibrium in the case under consideration can be written in the form

$$\int_0^\infty \sigma_{yy}^{\partial}(x'-x,\ y_0)\,\rho(x')\,dx' = \frac{1}{b}\left[S(x) + \sigma_{yy}^{e}(x)\right]. \tag{1}$$

Here $\sigma_{yy}^{\partial}(x'-x,\ y_0)$ are the stresses produced at point $(x,\ y_0)$ by a dislocation at point $(x',\ y_0)$; since the plate is assumed to be infinite in the direction of the x-axis, these stresses depend only on the difference $x'-x$. The remaining notation is as follows: $\sigma_{yy}^{e}(x)$ are stresses which would act at point $(x,\ y_0)$ in the absence of a crack; b is the Burgers vector of the dislocation; S(x) is the intermolecular attraction force.

Since $\sigma_{yy}^{\partial}(x'-x,\ y_0) \approx \mathrm{const}/(x'-x)$ as $(x'-x) \to 0$, the integral equation (1) is singular with respect to $\rho(x)$, and the integral is understood in the sense of its principal value.

After solving (1), we can determine the form of the crack from the relation

$$h(x) = b\int_0^x \rho(x)\,dx, \tag{2}$$

where h(x) is the height of the crack opening at a given point. We consider solutions of (1) limited at zero, since otherwise, in accordance with (2), the condition of the crack edges gradually closing at the crack mouth is not satisfied [2].

A necessary and sufficient condition for the existence of such a solution is [3] that the following relation be satisfied:

$$\int_0^\infty \omega(x)\left[S(x) + \sigma_{yy}^{e}(x)\right]dx = 0, \tag{3}$$

where the function $\omega(x)$ is a solution of a conjugate uniform equation corresponding to (1),

$$\int_0^\infty \omega(x')\,\sigma_{yy}^{\partial}(x-x',\ y_0)\,dx' = 0. \tag{4}$$

Equation (3) is equivalent to the condition of the vanishing of the coefficient of concentration of normal stresses produced near the crack tip [2]. In addition, the presence of a crack produces a concentration of tangential stresses of the type $T/(r)^{1/2}$, where r is the distance from the crack tip, and $T \neq 0$ for the asymmetrical disposition of the crack shown in Fig. 1. These stresses can produce tangential shear of the material along the extension of the crack plane, i.e., they can produce a transverse shear crack. However, it is henceforth assumed that apart from the "normal" there are "tangential" forces of molecular cohesion which are sufficiently large to prevent the formation of transverse shear cracks.

It should be noted that $\sigma_{yy}^{\partial}(t,\ y_0) = -\sigma_{yy}^{\partial}(t, y_0)$, so that Eq. (4) is a self-adjoint equation and $\omega(x)$ may be physically interpreted as the density of dislocations that form a crack in the absence of external loads.

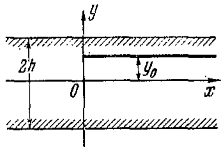

Fig. 1

If the function $\omega(x)$ is known, Eq. (3) makes it possible to analyze the crack propagation in relation to a variable external load. The main difficulty consists therefore in finding a solution of Eq. (4).

Although it is impossible to find this solution for the general anisotropic case, all the substantial singularities of the function $\omega(x)$ necessary for the analysis of crack propagation can be ascertained.

Formulas in [4] describing the kernel $\sigma_{yy}^{\partial}(t,\ y_0)$, show that all the length dimensions appear in the kernel in the form of a ratio to the plate thickness 2h and that this kernel contains the ratio y_0/h as a parameter. Consequently, after introducing the notation $\xi = \pi x/2h$ it may be asserted that the solution of Eq. (4) is in the form $\omega(x) \equiv \rho_0(\xi,\ y_0/h)$.

Moreover, since $\sigma_{yy}^{\partial}(t, y_0)$ as $t \to 0$ behaves as const/t, we have—as follows from the theory of singular integral equations—$\rho_0(\xi) \approx A/(\xi)^{1/2}$ as $\xi \to 0$, where $A = \mathrm{const} \neq 0$.

To study the behavior of the function $\rho_0(\xi)$ at large ξ, let us solve Eq. (4) by the Wiener-Hopf method. This method is used in the case under consideration because the kernel is a difference kernel and exponentially decreases at infinity [4]. As shown by Wiener and Peli [5], the asymptote $\rho_0(\xi)$ at infinity assumes the form

$$\rho_0(\xi) \approx \sum Q(\xi)\,e^{ik^*\xi}, \tag{5}$$

where the sum is taken over the k^* zeros of the Fourier transform which are in the analyticity band of this transform, and $Q(\xi)$ denote polynomials of degree $(n-1)$ where n is the multiplicity of the corresponding zero.

Let $\sigma(k)$ denote the Fourier transform of the function $\sigma_{yy}^{\partial}(t, y_0)$ with respect to t; $\sigma(k)$ can be easily obtained from formulas in [4]. Let us obtain an expression for $\sigma(k)$ only for the case of a medium with three mutually perpendicular planes of symmetry coinciding with the coordinate planes in Fig. 1 (a rhombic crystal or, in other terms, an orthotropic medium). In this case we have

$$\sigma(k) = -\frac{2id}{\sqrt{2\pi}\,(s_1-s_2)}\,\frac{\delta(k)}{\Delta(k)}, \tag{6}$$

$$\delta(k) = \begin{vmatrix} f_{11} & e^{ks_1h}-e^{ks_2h} & \mathrm{sh}\,ks_1(h-y_0) & \mathrm{sh}\,ks_2(h-y_0) \\ 0 & e^{-ks_1h}-e^{-ks_2h} & -\mathrm{sh}\,ks_1(h+y_0) & -\mathrm{sh}\,ks_2(h+y_0) \\ f_{31} & s_1e^{ks_1h}-s_2e^{ks_2h} & s_1\,\mathrm{ch}\,ks_1(h-y_0) & s_2\,\mathrm{ch}\,ks_2(h-y_0) \\ 0 & s_1e^{-ks_1h}-s_2e^{-ks_2h} & s_1\,\mathrm{ch}\,ks_1(h+y_0) & s_2\,\mathrm{ch}\,ks_2(h+y_0) \end{vmatrix}, \tag{7}$$

$$\Delta(k) = 4(s_1-s_2)^2(s_1+s_2)^2 \times \left[\frac{\mathrm{sh}^2 k(s_1+s_2)h}{(s_1+s_2)^2} - \frac{\mathrm{sh}^2 k(s_1-s_2)h}{(s_1-s_2)^2}\right],$$

$$f_{11} = s_2\,\mathrm{sh}\,ks_1(h-y_0) - s_1\,\mathrm{sh}\,ks_2(h-y_0),$$

$$f_{31} = s_1s_2\left[\mathrm{ch}\,ks_1(h-y_0) - \mathrm{ch}\,ks_2(h-y_0)\right]. \tag{8}$$

Here s_1, s_2, and d are values related in a certain way to other constants of the medium; d depends also on the Burgers vector [6, 7].

Simple calculations using expressions (6), (7), and (8) show that the point $k = 0$ contained within the analyticity band of the function $\sigma(k)$ is a triple zero of this function.

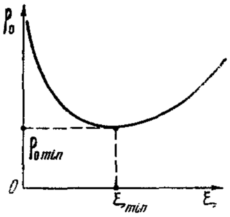

Fig. 2

It may therefore be concluded on the basis of (5) that $\rho_0(\xi)$ increases at infinity at a rate not slower than ξ^2. Such an asymptote $\rho_0(\xi)$ at infinity can be also obtained on the basis of the following simple physical considerations. Let us treat the breaking part of the plate as a beam fixed at the crack mouth and let h(x) denote the deflection of this beam. It is known (see for instance [1]) that this deflection satisfies an equation $h''''(x) = 0$, in the absence of external forces. A general solution of this equation will be a third degree polynomial; the coefficient of the highest degree of x in this polynomial is, generally speaking (i.e., for arbitrary boundary conditions) nonvanishing. At infinity we therefore have $h(x) \approx x^3$ and, consequently, (according to (2)), $\rho_0(x) \approx x^2$.

The foregoing considerations suggest that in the general anisotropic case $\rho_0(\xi) \approx \xi^2$ at infinity; however, this conclusion could be rigorously substantiated only by carrying out extremely tedious transformations.

Physical considerations also show that $\rho_0(\xi)$ should be of constant sign (positive to ensure determinacy). This is because $\rho_0(\xi)$ describes the distribution of dislocations in the absence of external forces. If the sign of $\rho_0(\xi)$ changed at a certain point, it would mean that dislocations of different signs are in equilibrium in the vicinity of this point; this is impossible since dislocations of different signs would attract and annihilate each other.

Finally, let us demonstrate the uniqueness of the solution of Eq. (4). As is known from the theory of the Wiener-Hopf method, every solution of Eq. (4) is in the form

$$\rho_0(\xi) = \frac{1}{2\pi i} \int_{\beta - i\infty}^{\beta + i\infty} P_n(u)\,\Phi_-(u)\,e^{u\xi}\,du,$$

where $\Phi_-(u)$ is a definite function and $P_n(u)$ is a certain polynomial

$$P_n(u) = a_n u^n + a_{n-1} u^{n-1} + \ldots + a_0 .$$

If the following notation is introduced :

$$f(\xi) = \frac{1}{2\pi i} \int_{\beta - i\infty}^{\beta + i\infty} \Phi_-(u)\,e^{u\xi}\,du,$$

$\rho_0(\xi)$ can be written in the form

$$\rho_0(\xi) = a_n f^{(n)}(\xi) + a_{n-1} f^{(n-1)}(\xi) + \ldots + a_0 f(\xi). \qquad (9)$$

On the other hand, as shown above, $\rho_0(\xi)$ as $\xi \to 0$ should have a singularity in the form $\mathrm{const}/(\xi)^{1/2}$. It is obvious that this singularity should be produced by a term with the higher derivative in (9), since otherwise the function $\rho_0(\xi)$ would have a more pronounced singularity at zero. If there were two or more linearly independent solutions in the form of Eq. (9), the coefficient of the higher derivative of function $f(\xi)$ could be made to vanish by constructing their linear combination (by an appropriate method) and the function obtained would not have the required singularity at zero. The contradiction obtained proves the uniqueness of the solution of Eq. (4).

It may therefore be postulated that the graph shown in Fig. 2 is an approximate representation of the function $\rho_0(\xi)$. This graph has one minimum, though the possibility of there being more than one minimum cannot be excluded a priori.

The above general considerations can be graphically illustrated on the example of a special case, in which it is possible to obtain an integral representation for the function $\rho_0(\xi)$. Let us consider the case in which $(s_1 + s_2)/(s_1 - s_2) = 2$, $y_0 = 0$, and let $s_1 - s_2 = s$, $ik = u$. In this case, accurate to an insignificant constant factor, we have

$$\sigma(u) = \mathrm{tg}^{3} 1/2\, hsu. \qquad (10)$$

Expression (10) is obtained from (6), (7), and (8) taking into account the above listed conditions.

It is possible to factorize the expression for $\sigma(u)$, as is required in the Wiener-Hopf method, and to obtain an integral representation for $\rho_0(\xi)$. Factorizing (10) with the aid of the Γ -function and then using the convolution theorem, we find

$$\rho_0(\xi) = Af'(\xi) + Bf(\xi), \qquad (11)$$

where

$$f(\xi) = \frac{d}{d\xi}\left[\iiint_D \frac{d\tau_1\,d\tau_2\,d\tau_3}{[(1 - e^{-\tau_1})(1 - e^{-\tau_2})(1 - e^{-\tau_3})]^{1/2}}\right]. \qquad (12)$$

In Eq. (11) A and B are certain constants; $A > 0$, $B \geq 0$. The integration interval D in (12) is a simplex determined by inequalities

$$\tau_1 \geqslant 0,\ \tau_2 \geqslant 0,\ \tau_3 \geqslant 0,\ \ \tau_1 + \tau_2 + \tau_3 \leqslant \xi .$$

Using expressions (11) and (12), one can easily see that $\rho_0(\xi) \sim 1/(\xi)^{1/2}$ as $\xi \to 0$ and $\rho_0(\xi) \sim \xi^2$ as $\xi \to \infty$, i.e., the behavior of this function at zero and at infinity is precisely as predicted by general considerations.

If only the general form of function $\rho_0(\xi)$ is known (Fig. 2), it is still possible qualitatively to describe the propagation of a crack along a plate in relation to an external load. To this end let us consider an important specific case in which $\sigma_{yy}{}^e(x) = P\delta(x - l)$, i.e., when the crack is kept open by concentrated forces P applied at a distance l from the crack mouth.

Let us consider the dependence of the crack "length" on the external load P. This relation is given in implicit form by Eq. (3) which in this case becomes

$$\int_0^\infty S(x)\,\rho_0\left(\frac{\pi x}{2h}\right)dx = P\rho_0\left(\frac{\pi l}{2h}\right). \qquad (13)$$

It should be noted that ρ_0 depends also on y_0/h and on the parameter. If it is borne in mind that $S(x) \neq 0$ only at small x [2] and if the behavior of $\rho_0(\xi)$ and the determination of the adhesion modulus M [2] are taken into account, Eq. (13) can be written in the form

$$A(y_0/h)\ M\ \sqrt{h} = P\rho_0\,(\pi l/2h,\ y_0/h). \qquad (14)$$

Equation (14), which gives the relation between P and l, should be solved graphically (Fig. 3). The curved and straight lines in Fig. 3 correspond, respectively, to the right and left hand sides of Eq. (14). Data in Fig. 3 show that two crack lengths correspond to each load: l_1 corresponding to a stable crack and l_2 to an unstable crack [2].

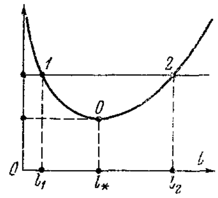

Fig. 3

Let us imagine the following experiment: an artificial crack (notch) is made in the plate after which concentrated load, increasing from zero at an infinitely slow rate is applied to the edge of the crack at a distance l from its mouth. The results will be as follows: if $l < l_*$ (in Fig. 3 we have $l = l_1$), no crack propagation will take place until the load reaches a level corresponding to l_1 (point 1). As the load is further increased, the crack will grow until its length reaches l_* at which rupture will take place. If, however, $l > l_*$ (in Fig. 3 we have $l = l_2$), no crack propagation will take place with increasing load, and rupture will take place when the load reaches a level corresponding to l_2.

The critical length l_* is found from the condition

$$\rho_0'(\pi l_* / 2h) = 0, \tag{15}$$

i.e., $l_* = C(y_0/h)h$. As was to be expected, the critical length at a given y_0/h is proportional to the plate thickness. The breaking load P_* is found from a relation which follows from (14)

$$P_* = \frac{A(y_0/h)}{\rho_0(\pi l_*/2h,\ y_0/h)} M\sqrt{h}. \tag{16}$$

A similar dependence of the breaking load on the plate dimensions was obtained in [2] by a method of dimensional analysis.

It should be noted that the abscissa and the ordinate of point 0 in Fig. 3 represent the critical length and the critical load.

If the load is not concentrated but applied along a certain length which is small in comparison with $(h - y_0)$, the qualitative nature of crack propagation in the plate will remain the same.

To ascertain the form of the critical load as a function of y_0/h, let us consider the equilibrium of an aggregate of screw dislocations whose distribution is also shown in Fig. 1. The problem is interesting in itself, since an aggregate of dislocations of this kind may be physically regarded as a twin [8] of a special type or as a longitudinal shear crack [9].

Proceeding as above, one can derive the equation of screw-dislocation equilibrium which, for the case of an isotropic medium, has the form

$$\int_0^\infty \rho(x')\left\{2\,\mathrm{cth}\frac{\pi(x'-x)}{4h} - \mathrm{th}\frac{\pi}{4h}[(x'-x)+2iy_0] - \mathrm{th}\frac{\pi}{4h}[(x'-x)-2iy_0]\right\}dx' = \frac{2\pi}{\mu b}[S(x)+\sigma_{yz}^e(x)], \tag{17}$$

where μ is the shear modulus.

Consideration of anisotropy in this case reduces to multiplying y_0 and h by a constant which depends on the elasticity moduli of the medium. The kernel of (17) can be obtained from formulas in [4]. A uniform equation corresponding to (17) can be also solved by the Wiener-Hopf method, in which case we obtain

$$\rho_0(\xi) = \frac{\alpha\beta}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\frac{\alpha^{-\alpha u}\beta^{-\beta u}\Gamma(\alpha u)\Gamma(\beta u)}{\Gamma(u)}e^{u\xi}du,$$

$$\xi = \pi x/2h,\quad \gamma > 0,\quad \alpha = {}^1/_2(1 - y_0/h),$$

$$\beta = {}^1/_2(1 + y_0/h),\quad \alpha+\beta = 1. \tag{18}$$

Let us examine certain properties of the function $\rho_0(\xi)$.

Equation (18) makes it possible to study the behavior of $\rho_0(\xi)$ at large and small ξ. In accordance with known theorems (see for instance [10])

$$\rho_0(\infty) = \lim_{u\to 0} uF(u),\qquad \rho_0(0) = \lim_{u\to\infty} uF(u). \tag{19}$$

Here F(u) is a Laplace transform of the function $\rho_0(\xi)$. In the case under consideration

$$F(u) = \alpha\beta\frac{\alpha^{-\alpha u}\beta^{-\beta u}\Gamma(\alpha u)\Gamma(\beta u)}{\Gamma(u)}.$$

In (19) it is assumed that limits on the right side exist.

Using the first formula of (19), we obtain that $\rho_0(\infty) = 1$.

Considerations analogous to those previously used show that $\rho_0(\xi) \sim 1/(\xi)^{1/2}$ as $\xi \to 0$, so that the second formula of (19) cannot be directly used. However, it can be applied to the difference between $\rho_0(\xi)$ and an arbitrary function which as $\xi \to 0$ behaves as $\mathrm{const}/(\xi)^{1/2}$. A function that can be conveniently used for this purpose has a form $A/(1 - e^{-2\xi})^{1/2}$. The Laplace transform of this function has the form

$$F_1(u) = \frac{A\sqrt{\pi}\,\Gamma({}^1/_2 u)}{\Gamma({}^1/_2 u + {}^1/_2)}.$$

Applying the second formula in (19) to the difference

$$\rho_0(\xi) - A/\sqrt{1-e^{-2\xi}},$$

and stipulating that this difference vanish as $\xi \to 0$, we find constant A. Thus, A is found from the condition

$$\lim_{u\to\infty} u\left[\alpha\beta\frac{\alpha^{-\alpha u}\beta^{-\beta u}\Gamma(\alpha u)\Gamma(\beta u)}{\Gamma(u)} - \frac{A\sqrt{\pi}\,\Gamma({}^1/_2 u)}{\Gamma({}^1/_2 u + {}^1/_2)}\right] = 0.$$

Removing the limit, we find

$$A = 2\sqrt{\alpha\beta} = \sqrt{1 - y_0^2/h^2}. \tag{20}$$

When the equilibrium of an aggregate of screw dislocations is considered, it is possible to prove that $\rho_0(\xi)$ is positive and monotonic by purely mathematical means without resorting to physical interpretation of this function.

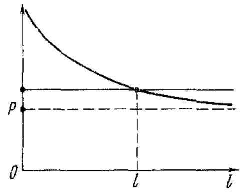

Fig. 4

To this end it should be noted that Eq. (18) can be rewritten in the form

$$\rho_0(\xi) = \frac{\alpha\beta}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} B(\alpha u, \beta u)\,e^{u[\xi - \alpha\ln\alpha - \beta\ln\beta]}du.$$

Here B(p, q) is the Euler beta-function. Using the known integral representation of the beta-function, one can write

$$\rho_0(\xi) = \frac{\alpha\beta}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\left[\int_0^1 t^{\alpha u-1}(1-t)^{\beta u-1}dt\right]e^{u[\xi-\alpha\ln\alpha-\beta\ln\beta]}du =$$

$$= \frac{\alpha\beta}{2\pi i}\int_0^1\frac{dt}{t(1-t)}\int_{\gamma-i\infty}^{\gamma+i\infty}\exp\left[u\left(\xi+\alpha\ln\frac{t}{\alpha}+\beta\ln\frac{1-t}{\beta}\right)\right]du =$$

$$= \alpha\beta\int_0^1\frac{dt}{t(1-t)}\delta\left(\xi+\alpha\ln\frac{t}{\alpha}+\beta\ln\frac{1-t}{\beta}\right).$$

Let us now use a known relation:

$$\delta(\psi(t)) = \sum_k\frac{\delta(t-t_k)}{|\psi'(t_k)|}.$$

Here t_k is the root of equation $\psi(t) = 0$.

It is easy to show that equation

$$\xi+\alpha\ln\frac{t}{\alpha}+\beta\ln\frac{1-t}{\beta} = 0,$$

has two roots within the interval (0,1); let these be denoted by $t_1(\xi)$ and $t_2(\xi)$ so that $t_1(\xi) \le \alpha$, $t_2(\xi) \ge \alpha$ and $dt_1/d\xi < 0$, $dt_2/d\xi > 0$. In this notation

$$\rho_0(\xi) = \alpha\beta\left(\frac{1}{|\alpha - t_1(\xi)|} + \frac{1}{|\alpha - t_2(\xi)|}\right).$$

The above expression shows (if the above cited properties of functions $t_1(\xi)$ and $t_2(\xi)$ are taken into account) that $\rho_0(\xi)$ is positive and monotonically decreasing. It should be noted that in the special case in which the twin (longitudinal shear crack) is in the middle of the plate, it is possible to calculate integral (18) in elementary functions. In this case

$$\rho_0(\xi) = 1 / \sqrt{1 - e^{-2\xi}}\,.$$

Let us again consider a specific case of an external load $\sigma^e_{yz}(x) = P\delta(x - l)$. Formulating a condition of orthogonality analogous to (3), we obtain (taking into account Eq. (20) and the behavior of $\rho_0(\xi)$ as $\xi \to 0$) the following equation describing the dependence of l on P:

$$\sqrt{1 - y_0^2 / h^2}\, M \sqrt{h} = P\rho_0 (\pi l / 2h)\,. \qquad (21)$$

A graphical solution of (21) is shown in Fig. 4, where the curve corresponds to the right side of Eq. (21) and the straight line to the left side. Analysis of Fig. 4, leads to the following conclusions regarding the growth of the twin under the influence of increasing P. When the load is increased, the twin length will gradually increase and, when the load reaches a certain level, the twin will occupy the entire length of the plate. This differs from the propagation of a normal fracture crack in that, as shown in Fig. 4, the concept of a critical length is meaningless in this case: the twin length will gradually increase, becoming infinite in the limit as $P \to P_*$. As for the breaking stress, it is given by

$$P_* = \sqrt{1 - y_0^2 / h^2}\, M \sqrt{h}\,. \qquad (22)$$

Equation (22) describes the dependence of P_* on y_0/h. It will be seen that as y_0 increases from zero to h, the breaking stress decreases from its maximum value $M(h)^{1/2}$ to zero. If $h \to \infty$ and $y_0 \to \infty$ but with $(h - y_0) = a$, where a is constant, we have

$$P_* \to M \sqrt{2a}\,.$$

It should be pointed out that the adhesion moduli for a twin and a crack are, generally speaking, different.

The dependence of the breaking stress on the distance between a crack and the plate boundary in the case considered above for a normal fracture crack will be analogous to (22).

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